Lecture 2: Review Part 1 Fundamentals of Hydraulics

WMD651: Water Resources Systems Design

2021 January 11

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Lecture Overview

- **Purpose:** review fundamental concepts for hydraulics, including:
    - Conservation laws – mass, energy, momentum
    - HGL and EGL
    - Bernoulli principle
    - Energy equation and head losses
- Subsequent lecture:
    - Head loss formulae – friction and local losses
    - Moody diagram for friction factor
    - Pumps – basis, performance curves
    - System curves

Conservation Laws

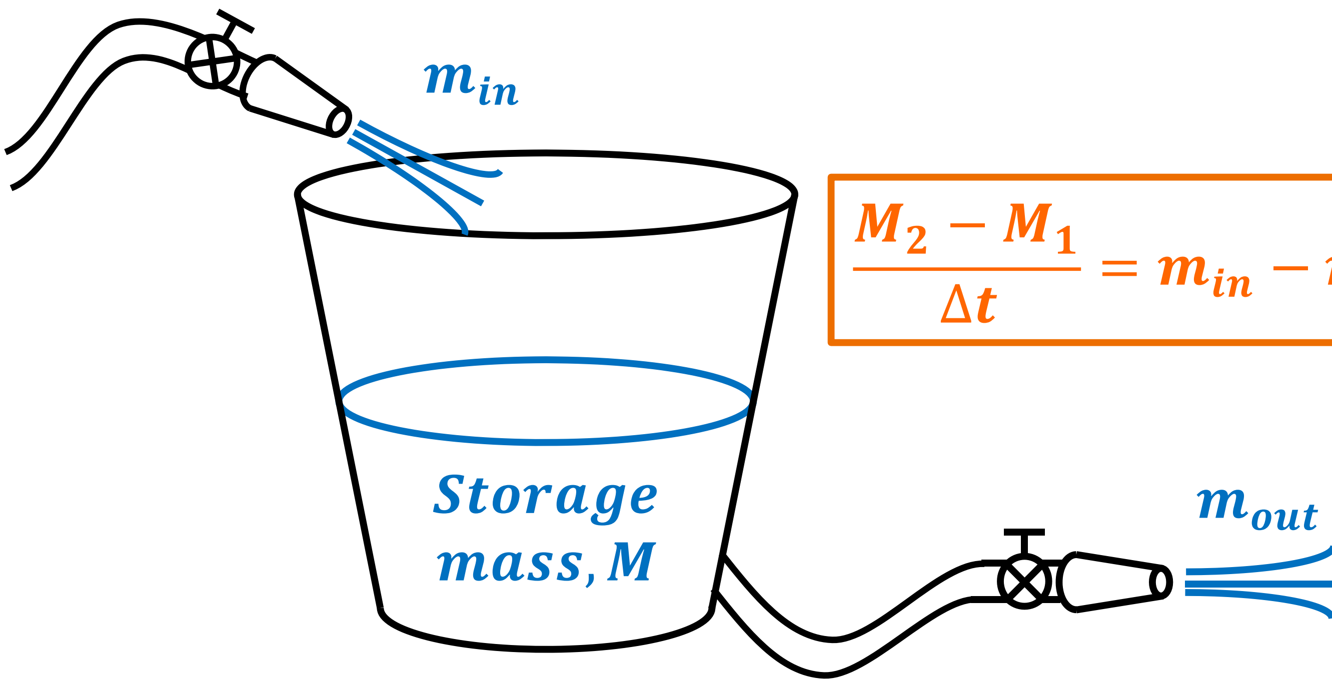
- Describe conservation (accounting) of **physical quantities**
- Govern behaviour of physical systems
- Form the basis of numerical models
- A few key relations include:
  - Conservation of **mass**
  - **Continuity** equation
  - Conservation of **energy**
  - Conservation of **momentum**

Conservation of Mass

- Mass conservation equation:

[Mass change over time] = [mass rate in] – [mass rate out]

- Can be applied to control volumes with or without **storage** → without storage, [rate in] = [rate out]



$$\frac{M_2 - M_1}{\Delta t} = m_{in} - m_{out}$$

Mass Conservation Example 1

- **Problem:** Consider a sink with Q_{in} = 0.02 m^3/s, Q_{out} = 0.019 m^3/s, and an initial volume of $\forall_1$ = 0.01 m^3. (a) How fast is the volume changing? (b) When will the volume reach a volume of $\forall_2$= 0.02 m^3?

Mass Conservation Example 1

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- **Part (a) solution** – apply mass conservation eq.:
  - $\frac{d\forall}{dt} = Q_{in} - Q_{out}$
    [rate of change in mass] = [mass rate in] – [mass rate out]
  - $= (0.02\ m^3/s) - (0.019\ m^3/s)$
  - $= +0.001\ m^3/s$ → *volume is increasing (sink filling)*

Mass Conservation Example 1

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- **Part (a) solution** – apply mass conservation eq.:
  - $\frac{d\forall}{dt} = Q_{in} - Q_{out}$
  - $= (0.02\, m^3 s) - (0.019\, m^3 s)$
  - $= +0.001\, m^3/s$ → *volume is increasing (sink filling)*
- **Part (b) solution:**
  - $\frac{d\forall}{dt} \approx \frac{\forall_2 - \forall_1}{\Delta t} = 0.001\, m^3/s$
  - So: $\Delta t = \frac{\forall_2 - \forall_1}{0.001\, m^3/s} = \frac{(0.02\, m^3) - (0.01\, m^3)}{0.001\, m^3/s} = 10\ s$

Mass Conservation Example 2

- **Problem:** Consider a sealed mixing reactor with volume V = 10 m^3, inflow concentration C_{in} = 0.1 kg/m^3, and initial concentration C_0 = 0 kg/m^3. Given a constant flow of Q = 0.5 m^3/s, what is the outflow concentration over time?

Mass Conservation Example 2

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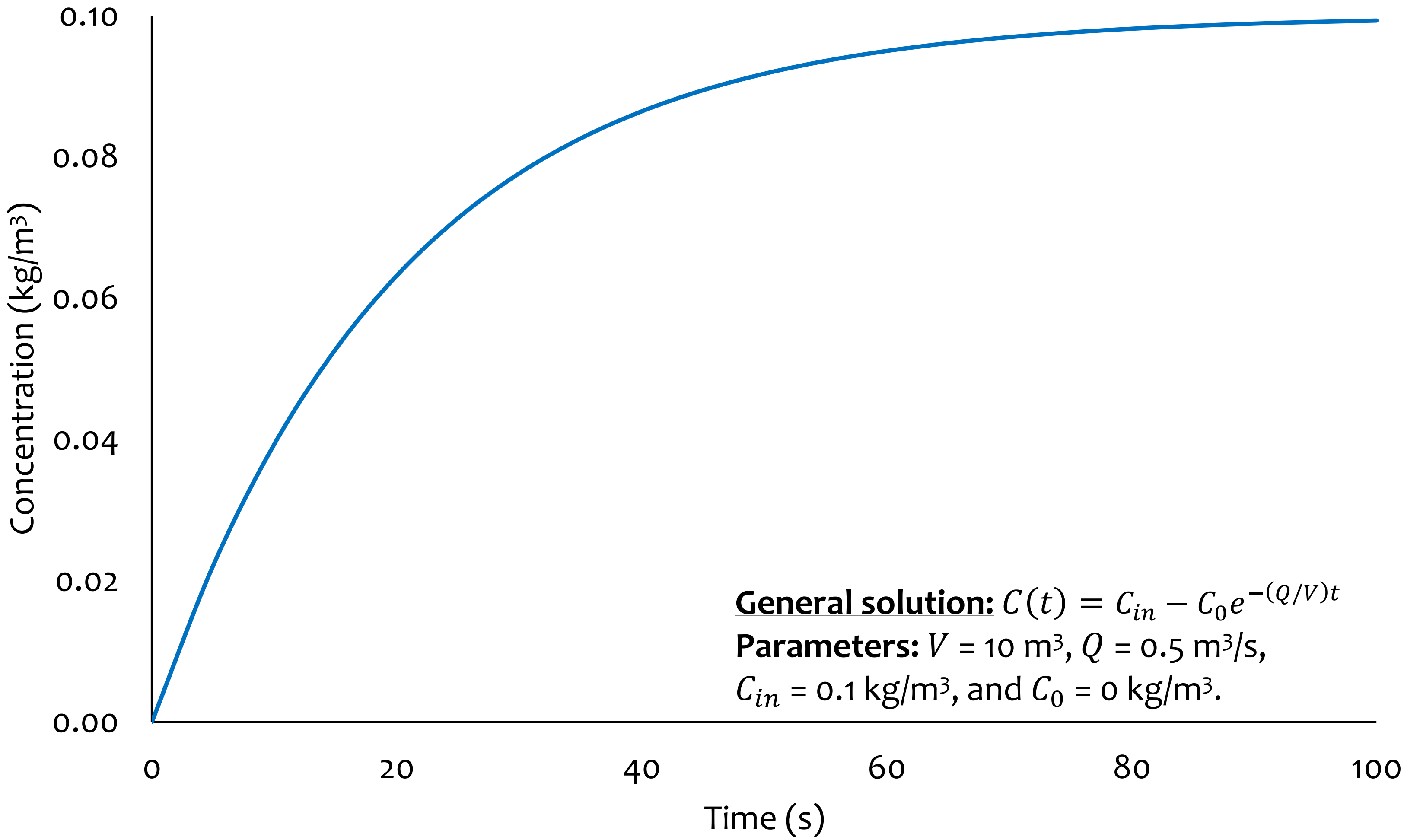
- **Solution:**
  - Reactor contaminant mass: $M = V \cdot C$
  - Contaminant inflow rate: $m_{in} = Q \cdot C_{in}$
  - Contaminant outflow rate: $m_{out} = Q \cdot C$
  - Assemble mass conservation equation:
    - $\frac{dM}{dt} = m_{in} - m_{out}$
    - $V\frac{dC}{dt} = C_{in}Q - CQ$ → first order differential equation

Mass Conservation Example 2

- **Solution (continued):**
  - Solving for $C(t)$ gives: $\boldsymbol{C(t) = C_{in} - C_0 \cdot e^{-(Q/V)t}}$
  - Note: *you do not need to know how to solve differential equations in WMD651*
  - Equation represents exponential growth in contaminant outflow up to equilibrium
    - Initially, $C(t)$ = 0
    - As contaminant enters the reactor, it mixes, increases the contaminant concentration, and then leaves the reactor in a diluted state
    - However, over time $C(t)$ approaches C_{in} → once equilibrium is reached, no more dilution!
  - What does the solution look like?

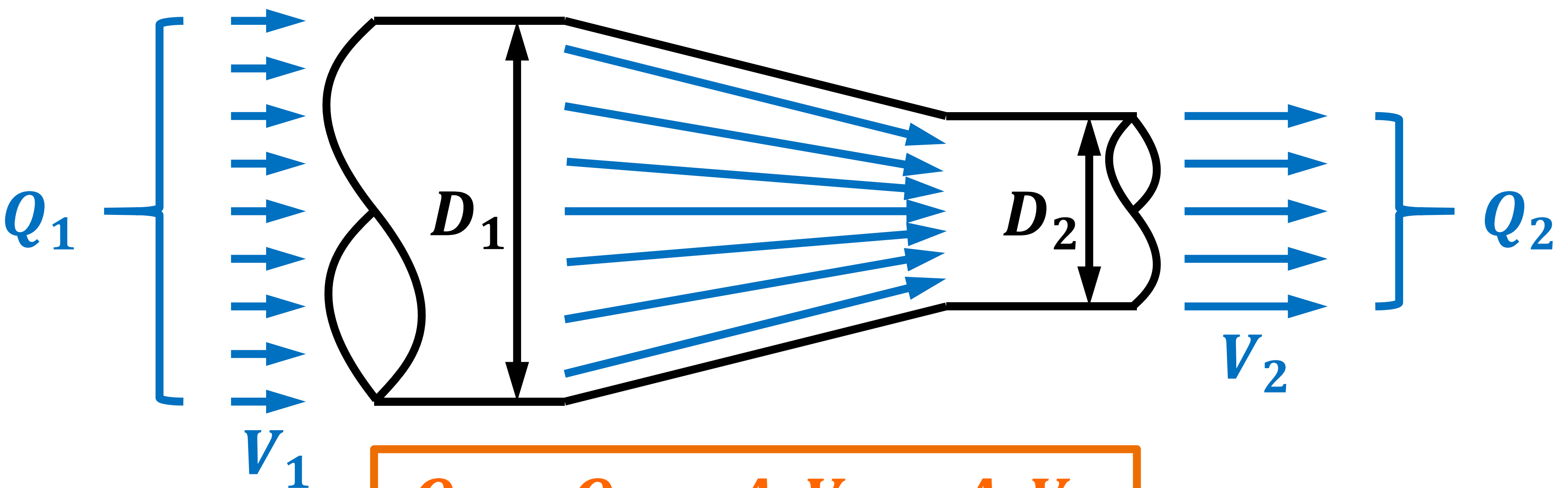
Mass Conservation Example 2

- **Solution (continued):**



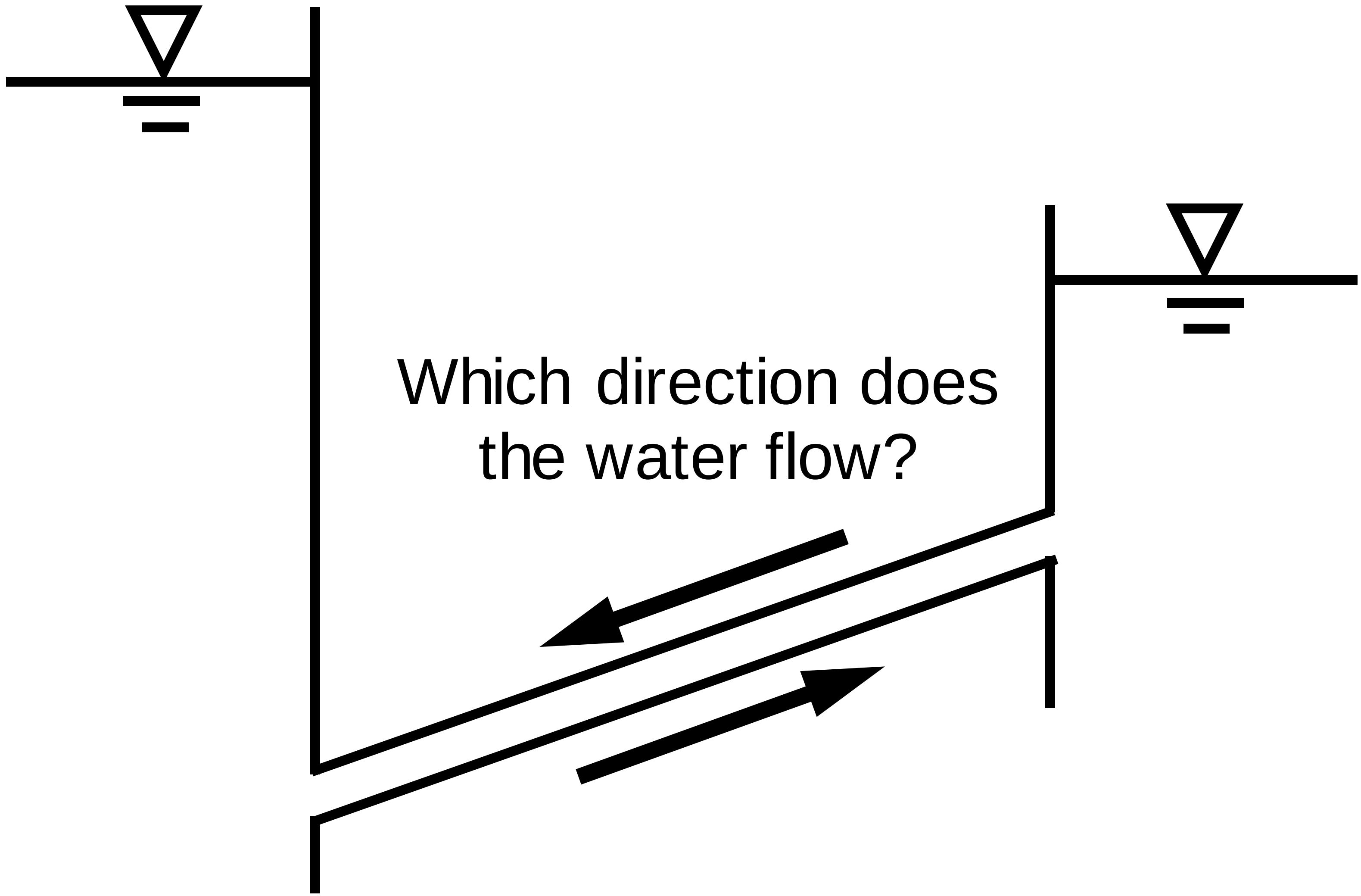
Continuity Equation

- What about control volumes with no storage?
- Can apply continuity equation: Q=V*A

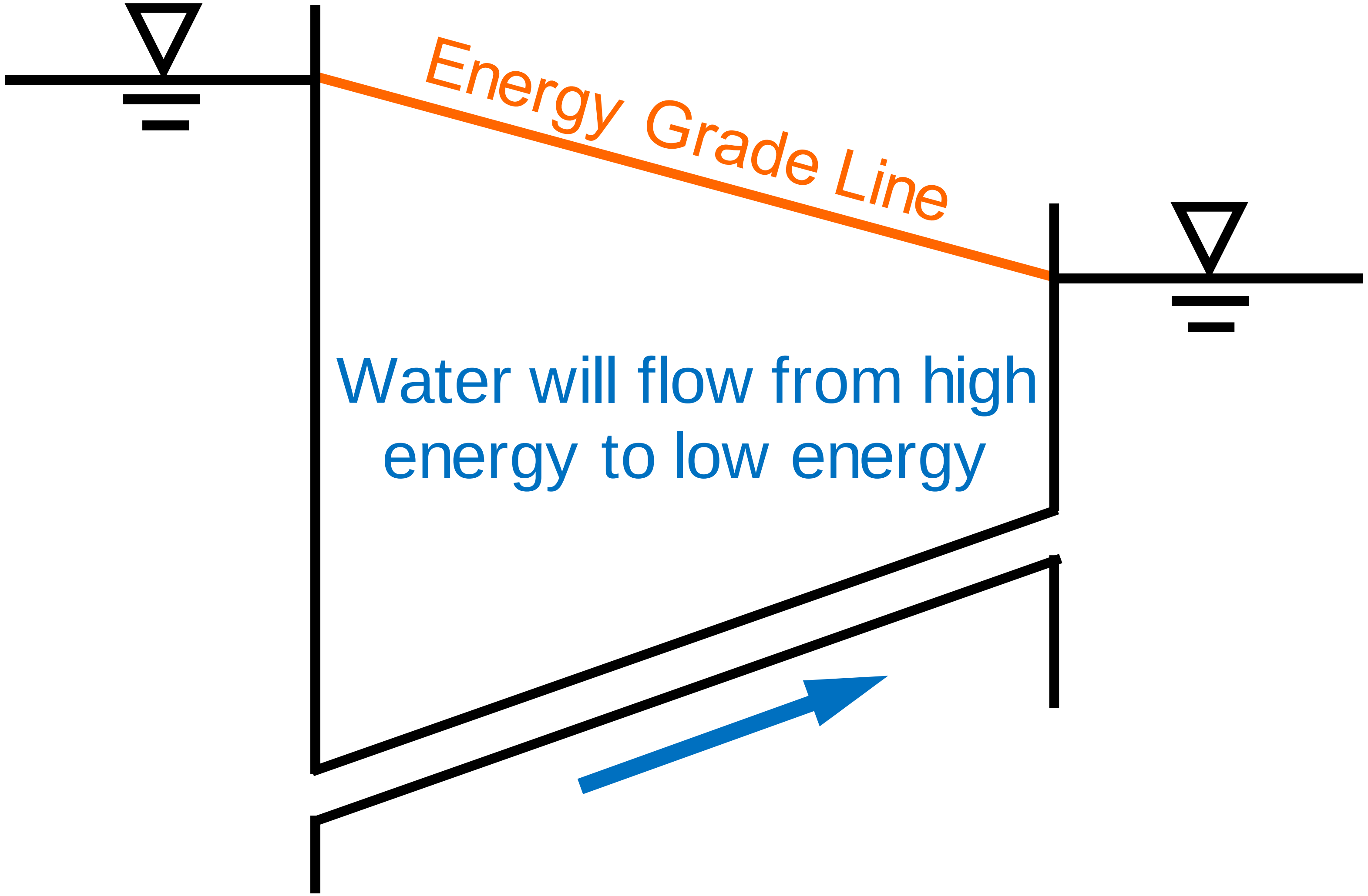


$$Q_1 = Q_2 \rightarrow A_1 V_1 = A_2 V_2$$

Which way does water flow?

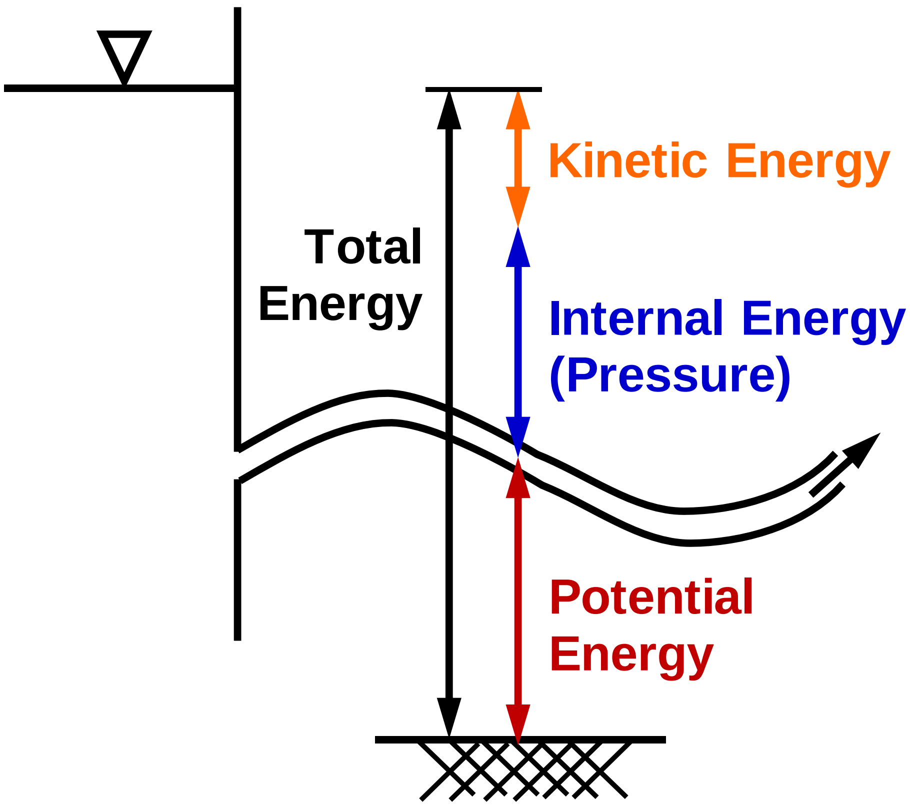


Which way does water flow?



Characterizing Energy

- Three energy components:
  - Kinetic energy
  - Internal (pressure) energy
  - Potential (elevation) energy



Total energy:

$$E = Z + \frac{P}{\gamma} + \frac{V^2}{2g}$$

E = total energy (m)
Z = elevation (m)
P = pressure (kPa)
V = velocity (m/s)
γ = 9.81 kN/m^3
g = 9.81 m/s^2

What does it mean?

$$E = Z + \frac{P}{\gamma} + \frac{V^2}{2g} = \mathrm{Z} + \frac{\mathrm{P}}{\mathrm{y}} + \frac{\mathrm{Q}^2}{2\mathrm{gA}^2}$$

- E = amount of energy per unit of fluid
- Z = elevation, a measure of potential energy
  - At higher elevation, there is higher potential energy!
- P/γ = internal pressure energy
- $V^2/2g$ = kinetic energy (velocity head)
  - Higher V → much higher kinetic energy, since kinetic energy is proportional to velocity squared

Hydraulic and Energy Grade Lines

- Hydraulic grade line:
  - Sum of internal and potential energies
  - Equation for head:

$$H = Z + P/\gamma$$

  where Z = elevation (m), P = pressure (kPa), and γ = specific weight of water (9.81 kN/m^3)

- Energy grade line:
  - Sum of kinetic, internal, and potential energies
  - Equation for energy:

$$E = Z + P/\gamma + V^2/2g$$

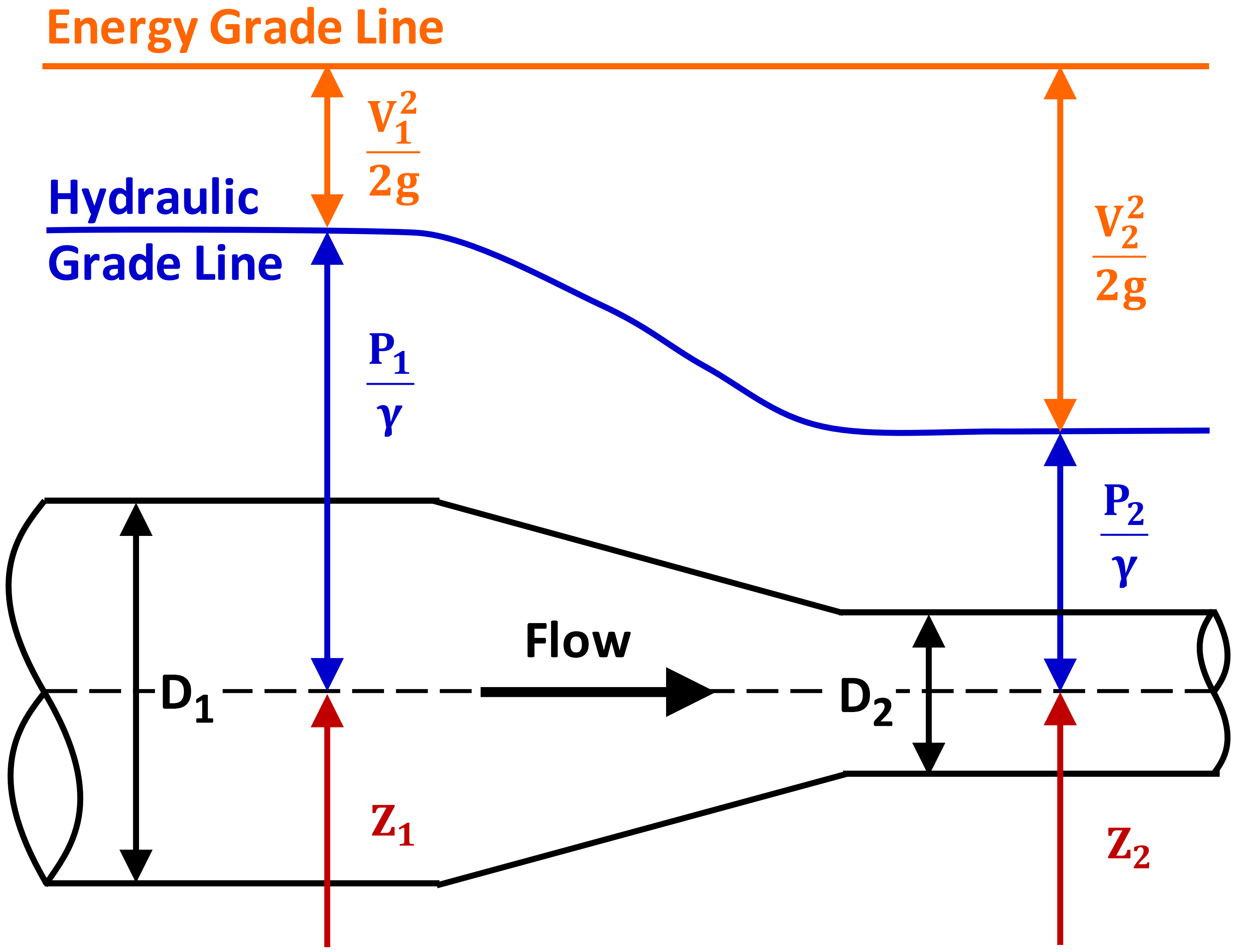
  where V = velocity (m/s), g = 9.81 m/^{2}

Bernoulli Principle

- The Bernoulli principle encompasses EGLs
- Represents energy conservation… **but with the assumption of no energy losses!**
- Provides a reasonable approximation when energy losses are negligible
- Bernoulli equation:

$$E_1 = E_2 \rightarrow Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

Bernoulli Principle



Conservation of Energy

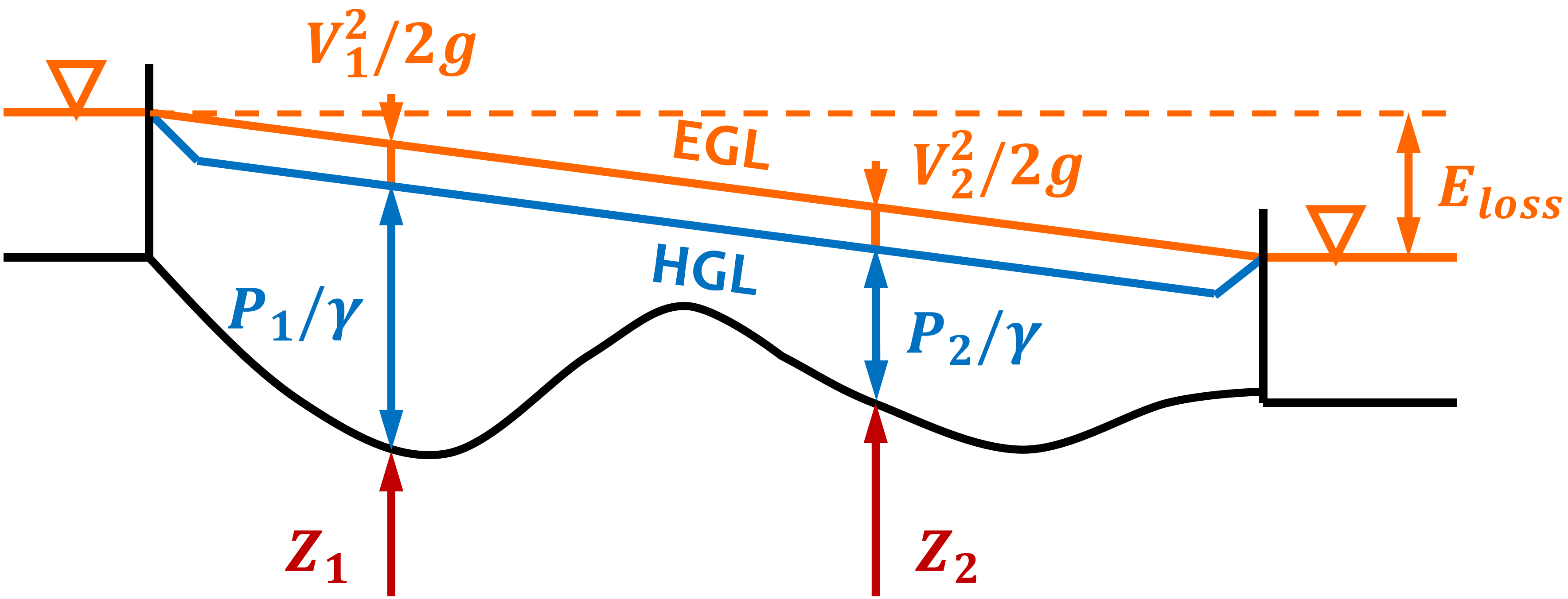
- Bernoulli equation only valid for small energy (head) losses → **what if losses are not small?**
- When losses are no longer small, they cannot be ignored from our energy expression…
- Energy equation:

$$E_1 + E_{gain} = E_2 + E_{loss}$$

where E_{gain} = the amount of energy (m) gained from an external source (e.g., from a pump), and E_{loss} = energy lost (m) through energy dissipation (friction, turbulence)

Conservation of Energy

- Consider a constant diameter pipe between two water sources:



$$E_1 = Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} \qquad E_2 = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

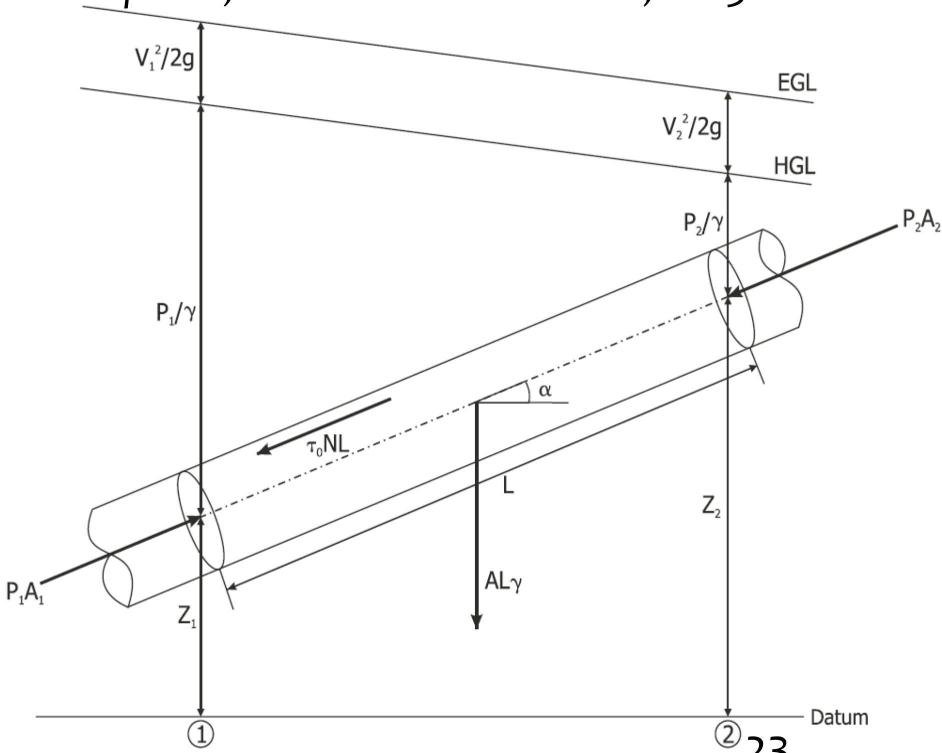
A Quick Note

- ‘Energy losses’ and ‘head losses’ are common terms in hydraulics and fluid dynamics
- But… energy cannot be created nor destroyed!
- The term ‘losses’ is thus something of a misnomer
- By ‘energy/head loss’, we actually mean that hydraulic (mechanical) energy is converted into thermal energy

Conservation of Momentum

- HGLs and EGLs are useful, but we know energy losses may be important → **how important?**
- Conservation of momentum can be used to derive expression for head losses:
  - Consider forces on fluid
  - Balance forces for:
    - Pressure
    - Potential (elevation)
    - Friction
  - Derivation not covered

Chapter 2, Haestad Methods et. al, 2003



Concluding Notes

- First principles provide foundation for developing understanding of more complex mechanics
- Most importantly, provide knowledge of basis for developing and using numerical models